



ChE-309 TP-4

Fixed and fluidized beds

Instructions for use, spring 2025



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1. General objective

A Swiss company in the field of functionalized silica powder has hired you to determine the power of the pump that will bring water at 25°C into a fluidized bed of silica particles (spherical beads, 200 kg). In the bed, the beads undergo surface functionalization and must be suspended, or fluidized, during processing to ensure total surface coverage. If functionalization is not effective, surface impurities could compromise the polarity and separating power of the product when used for chromatographic purposes. The company provided you with particle samples but did not provide information on the characteristics of the particles. First you need to characterize the particles, and then you need to characterize their fluidization properties to be able to determine the size of the pump needed.

2. Theoretical basis.

2.1. Definition of the drag coefficient around submerged objects

2.1.1. Introduction and types of drag.

The flow of a fluid around a submerged body occurs in many chemical engineering and other process applications. This takes place, for example, around particles in suspension, through a drying or filtering membrane, through heat exchangers, etc. It is useful to be able to predict friction losses and/or the force on immersed objects in these various applications.

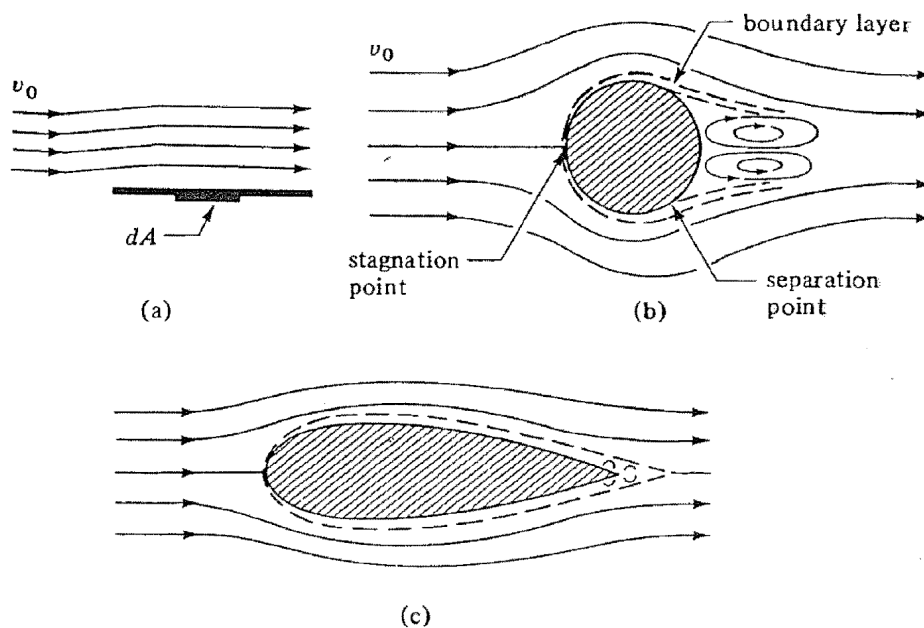


Figure 1: Flow of fluid around different submerged bodies

In the Figure 1 above (part a) the fluid flow is parallel to the smooth surface of the flat, solid plate and the force F (in Newton) on a surface element dA [m^2] of the plate is the shear stress τ_w multiplied by dA , i.e. $\tau_w dA$. The total force is the sum of the integrals of these quantities evaluated over the entire surface of the plate. Here the transfer of moment to the surface produces a tangential stress or surface slip.

In many cases, however, the submerged body is a solid that has different angles to the direction of fluid flow. As shown in Figure 1b the velocity v_0 is uniform as the fluid approaches the suspended body in a large tube. The lines, called stream lines, represent the path of fluid elements around the suspended body. The thin boundary layer adjacent to the solid surface is represented by a dashed line and at the edge of this layer the velocity is substantially the same as the velocity of the adjacent fluid. At the front center of the body, at a place called the stagnation point, the velocity of the fluid is zero. The growth of the boundary layer starts from this point and will continue over the entire surface of the object until it separates. Due to the velocity gradient in the boundary layer a tangential stress is created. This is called friction drag. Outside the boundary layer the direction of the fluid changes direction to pass around the solid, it will be accelerated in front of the solid and slowed down afterwards. Due to these effects, additional force is exerted by the fluid on the body. This phenomenon, called pressure drag (or form drag), is in addition to the friction drag in the boundary layer.

As shown in Figure 1b, the boundary layer detachment occurs and a wake covering the entire back of the object occurs when large vortices are present, contributing to pressure drag. The separation point depends on the shape of the particle, the Reynolds number, and so on.

Pressure drag can be minimized by streamlining the body (Figure 1c) to shift the layer detachment to the back of the body, which greatly reduces the size of the wake.

2.1.2. Coefficient of drag.

From the previous discussion, it is evident that the geometry of the submerged solid is a major factor in determining the amount of total drag force exerted on the body. Correlations based on geometry and flow characteristics for solid objects suspended or retained in a flow (submerged objects) are similar in concept to the correlation between friction factor and Reynolds number given for flows in pipes. Through pipes, the coefficient of friction is defined as the ratio between the tensile force per unit area (shear stress) and the product of velocity and density as given in equation 1:

$$f = \frac{\tau_s}{\rho v^2/2} = \frac{\Delta p_D \pi R^2}{2\pi R \Delta L} \cdot \left(\frac{\rho v^2}{2}\right)^{-1} \quad (\text{eq. 1})$$

where:

f = Fanning friction factor

Δp_D = pressure loss due to drag

πR^2 = cross-sectional area

$2\pi R \Delta L$ = wetted surface

Similarly, for flow around submerged objects, the drag coefficient C_D is defined as the ratio of the total drag force per unit area and $\rho v_0^2/2$:

$$C_D = \frac{F_D}{A_p} \cdot \left(\frac{\rho v_0^2}{2} \right)^{-1} \quad (\text{eq. 2})$$

where:

F_D = total force of drag [N]

A_p = area obtained by projecting the body on a plane perpendicular to the flow lines [m²]

C_D = coefficient of drag [dimensionless]

v_0 = fluid approach speed [m/s]

ρ = density of the fluid [kg/m³]

By solving equation 2 for the total drag force, we get:

$$F_D = C_D \frac{v_0^2}{2} \rho A_p \quad (\text{eq. 3})$$

The Reynolds number for a given solid immersed in a flowing liquid is:

$$N_{Re} = \frac{D_p v_0 \rho}{\mu} = \frac{D_p G_0}{\mu} \quad (\text{eq. 4})$$

where $G_0 = v_0 \rho$

2.2. Flow around a sphere, a long cylinder, or a disc

There is a relationship between C_D and N_{Re} for each case of immersed object, determined experimentally. These are shown in Figure 2 for spheres, long cylinders, and discs. In these cases, the disc face and the cylinder axis are always considered perpendicular to the direction of flow. However, in the laminar region, i.e., for low Reynolds numbers (less than about 1.0), the drag force determined experimentally is the same as the Stokes' Law equation as follows:

$$F_D = 3\pi\mu D_p v_0 \quad (\text{eq. 5})$$

By combining equations 3 and 5 and solving for C_D , the Stokes' law drag coefficient is:

$$C_D = \frac{24}{D_p v_0 \rho / \mu} = \frac{24}{N_{Re}} \quad (\text{eq. 6})$$

The variation of C_D with respect to N_{Re} is quite complicated due to the interaction of factors that control layer strength and pressure drag. For a sphere, when the Reynolds number is increased beyond the range of Stokes' law, detachment of the boundary layer occurs. If N_{Re} is further increased, the separation point shifts and C_D decreases. In the region of N_{Re} of about 1×10^3 to 2×10^5 , the drag coefficient is approximately constant at $C_D = 0.44$ for a sphere. From $N_{Re} = 3 \times 10^5$ there is a sudden drop of C_D which is the result of the complete turbulence in the boundary layer and the displacement of the separation point downstream. Above a N_{Re} of about 5×10^5 drag coefficients are again approximately constant: 0.13 for a sphere, 0.33 for a cylinder and 1.12 for a disk.

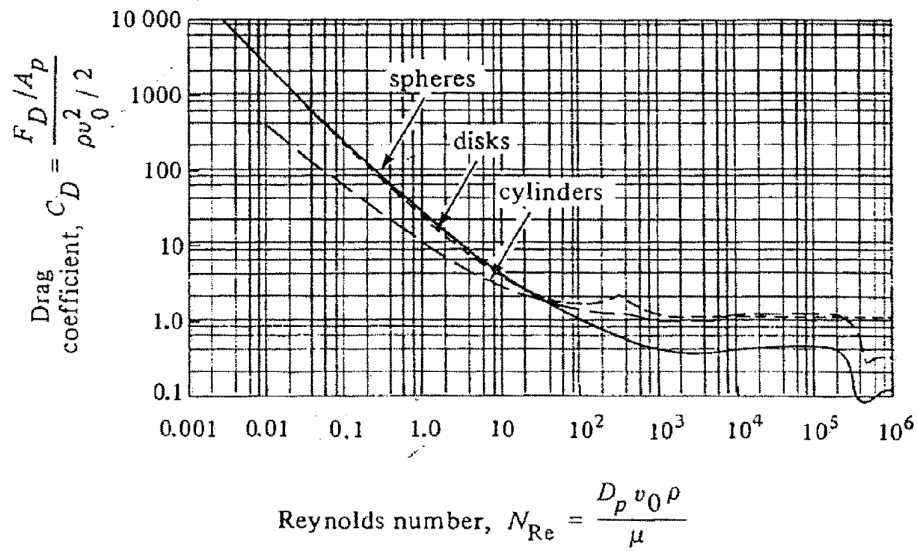


Figure 2: Correlations between the Reynolds number and the drag coefficient for spheres, disks, and cylinders

The flow of fluids in cylinders or tubes takes place in heat exchangers or other process applications. The tube banks can be arranged in a number of different geometries. Due to the many possible configurations, it is not possible to have a single correlation for pressure drops and friction factors.

2.3. Flow through a fixed (stationary) bed

2.3.1. Introduction.

A system of considerable importance in the chemical industry and other areas of process engineering is the packed bed or packed column which is used in fixed bed catalytic reactors, solute adsorption or absorption, filter beds, etc. Bed filler particles can be spheres, irregular particles, cylinders, or various types of commercial packing. In the discussion that follows, it is assumed that the filling is uniform everywhere in the column or bed. The ratio of column diameter to packing diameter should be a minimum of 8:1 to 10:1 to be able to neglect the wall effects. In the theoretical approach used, the packed column is considered to be a bundle of twisted tubes with a different cross-section. The theory developed for simple straight tubes is used to develop the results for the bundle of twisted tubes.

2.3.2. Laminar flow in packed beds.

Some definitions are needed for the flow in a particle bed such as the porosity ε in a packed bed:

$$\varepsilon = \frac{\text{empty volume in the bed}}{\text{total volume of bed (void plus solid)}} \quad (\text{eq. 7})$$

The specific surface area of a particle a_v in $[\text{m}^{-1}]$:

$$a_v = \frac{s_p}{v_p} \quad (\text{eq. 8})$$

where S_p is the surface area of the particle in $[m^2]$ and V_p is the volume of the particle in $[m^3]$. For a spherical particle:

$$a_v = \frac{6}{D_p} \quad (\text{eq. 9})$$

where D_p is the particle diameter in $[m]$. Since $(1 - \varepsilon)$ is the volume fraction of the particles in the bed, we can write:

$$a = a_v(1 - \varepsilon) = \frac{6}{D_p}(1 - \varepsilon) \quad (\text{eq.10})$$

where a is the ratio of the total area in the bed to the total volume of the bed (void volume as well as the volume of particles) in $[m^{-1}]$.

The mean interstitial velocity of the fluid in the bed is v $[m/s]$. It is related to the superficial velocity, v' , based on the cross-section of the empty container by:

$$v' = \varepsilon v \quad (\text{eq.11})$$

The hydraulic radius r_H is defined as the ratio between the cross-sectional area of the channel (the path the fluid takes through the bed) and the wetted perimeter of the flow channel. Thus, in the case of turbulent flow only:

$$r_H = \frac{(\text{flow cross section})}{(\text{wetted perimeter})} = \frac{(\text{empty volume in bed})}{(\text{total wetted surface of solids})} = \frac{\text{volume empty/volume total}}{\text{surface area wetted/volume total}} = \frac{\varepsilon}{a} \quad (\text{eq.12})$$

By combining equations 10 and 12:

$$r_H = \frac{\varepsilon}{6(1-\varepsilon)} D_p \quad (\text{eq.13})$$

Since the equivalent diameter of a channel is $D = 4r_H$, the Reynolds number for a packed bed is as follows using equations 11 and 13:

$$N_{Re} = \frac{(4r_H)v\rho}{\mu} = \frac{4\varepsilon}{6(1-\varepsilon)} D_p \frac{v'}{\varepsilon} \frac{\rho}{\mu} = \frac{4}{6(1-\varepsilon)} \frac{D_p v' \rho}{\mu} \quad (\text{eq.14})$$

For packed (stationary) beds, Ergün defined the Reynolds number as above but without the term $4/6$.

$$N_{Re,p} = \frac{D_p v' \rho}{(1-\varepsilon)\mu} = \frac{D_p G'}{(1-\varepsilon)\mu} \quad (\text{eq.15})$$

where $G' = v' \rho$.

For laminar flow, the Hagen-Poiseuille equation defines the pressure drop:

$$\Delta p_f = (p_1 - p_2)_f = \frac{32\mu v(L_2 - L_1)}{D^2} \quad (\text{eq.16})$$

where:

p_1 = pressure at point 1 [N/m²]

p_2 = pressure at point 2 [N/m²]

v = average fluid velocity in the tube [m/s]

D = internal diameter [m]

$(L_2 - L_1) = L_l$ = length of the bed [m]

$(p_1 - p_2) = \Delta p_f$ = pressure drop due to friction

Combining this equation with the 13 for r_H and eq. 11, we find:

$$\Delta p_f = \frac{32\mu v L_l}{D^2} = \frac{32\mu \left(\frac{v'}{\varepsilon}\right) L_l}{(4r_H)^2} = \frac{(72)\mu v' L_l (1-\varepsilon)^2}{\varepsilon^3 D_p^2} \quad (\text{eq.17})$$

The actual distance travelled by the liquid in the bed is greater than L_l due to the tortuous trajectory of the liquid through the bed and the use of the hydraulic radius foresees a too big velocity v . Experimental data show that the constant should be 150 instead of 72, giving the Blake-Kozeny equation for laminar flow in a packed bed, with a void fraction less than 0.5, an effective particle diameter D_p and when $N_{Re,p} < 10$

$$\Delta p_f = \frac{150\mu v' L_l (1-\varepsilon)^2}{D_p^2 \varepsilon^3} \quad (\text{eq.18})$$

2.3.3. Turbulent flow in packed beds.

For a turbulent flow, we use the same procedure, starting with the equation below:

$$\Delta p_f = 4f\rho \frac{L_l}{D} \frac{v^2}{2} \quad (\text{eq.19})$$

And substituting in equations 11 and 13 we can get:

$$\Delta p_f = \frac{3f\rho(v')^2 L_l}{D_p} \frac{1-\varepsilon}{\varepsilon^3} \quad (\text{eq.20})$$

For a very turbulent flow the coefficient of friction should approach a constant value (as it does for the classic submerged objects shown in Figure 2). In addition, it is assumed that all packed beds should have the same relative roughness (in the limiting case). Experimental data indicate that $3f = 1.75$. Therefore, the final equation for a turbulent flow for $N_{Re,p} > 1000$, which is called the Burke-Plummer equation, becomes:

$$\Delta p_f = \frac{1.75\rho(v')^2 L_l}{D_p} \frac{1-\varepsilon}{\varepsilon^3} \quad (\text{eq.21})$$

Adding up equation 18 for laminar flow and equation 21 for turbulent flow, Ergün proposed the following general equation for low, intermediate, and high Reynolds numbers that was experimentally tested:

$$\Delta p_f = \frac{150\mu v' L_l (1-\varepsilon)^2}{D_p^2 \varepsilon^3} + \frac{1.75\rho(v')^2 L_l (1-\varepsilon)}{D_p \varepsilon^3} \quad (\text{eq.22})$$

Rewriting equation 22 in terms of dimensionless groups gives:

$$\frac{\Delta p_f \rho D_p \varepsilon^3}{(G')^2 L_l (1-\varepsilon)} = \frac{150}{N_{\text{Re},p}} + 1.75 \quad (\text{eq.23})$$

This is called the Ergün equation and can be used for liquids or for gases using the density ρ of the gas at the arithmetic mean of the inlet and outlet pressure. The velocity v' changes in the bed for a compressible fluid, but G' is constant. For high values of $N_{\text{Re},p}$ equations 22 and 23 are reduced to equation 21 and are reduced to equation 18 for low values.

2.3.4. Form factors.

Many particles in packed beds are often irregularly shaped. The equivalent particle diameter is defined as the diameter of a sphere having the same volume as the particle. The sphericity factor (or form factor) ϕ_s of a particle is the ratio of the surface of this sphere having the same volume as the particle to the effective surface of the particle. For a sphere, the surface $S_p = \pi D_p^2$ and the volume is $V_p = \pi D_p^3/6$. Therefore, for each particle, $\phi_s = \pi D_p^2/S_p$, where S_p is the effective surface area of the particle and D_p is the particle diameter (effective diameter) of the sphere having the same volume as the particle. So:

$$\frac{S_p}{V_p} = \frac{\pi D_p^2/\Phi_S}{\pi D_p^3/6} = \frac{6}{\Phi_S D_p} \quad (\text{eq.24})$$

From equation 8:

$$a_v = \frac{S_p}{V_p} = \frac{6}{\Phi_S D_p} \quad (\text{eq.25})$$

Finally, equation 10 becomes:

$$a = \frac{6}{\Phi_S D_p} (1 - \varepsilon) \quad (\text{eq.26})$$

For a sphere, $\phi_s = 1,0$, for a cylinder whose diameter = length ϕ_s is fixed at 0.874 and for a cube ϕ_s is 0.806. For granular materials, it is difficult to measure the actual volume and surface area in order to obtain the equivalent diameter. Therefore, D_p is generally considered to be the nominal size determined from visual analysis. The surface is determined by adsorption measurements or measurements of the pressure drop in a particle bed. Finally, equation 24 is used to calculate ϕ_s . Typical values for many crushed materials are between

0.6 and 0.7. For the convenience of the cylinder and cube, the nominal diameter is sometimes used (instead of the equivalent diameter) which then gives a form factor of 1.0.

2.4. Flow rate in fluidized beds

2.4.1. Minimum fluidization velocity and porosity.

When a fluid flows upward through a particle packed bed, the particles remain stationary at low velocity. With increasing fluid velocity, the pressure drop increases according to the Ergün relation (eq. 22 or 23). At some point, the force of pressure loss multiplied by the cross-sectional area is equal to the force of gravity on the mass of the particles. Then the particles begin to move, and this is the beginning of fluidization or the minimal fluidization point. The fluid velocity at which fluidization starts is the minimum fluidization velocity, v'_{mf} in m/s based on the cross-section of the empty column (superficial velocity).

The porosity of the bed at the time true fluidization occurs is the minimum fluidization porosity and is ε_{mf} . The bed develops this degree of porosity before particle movement occurs. This minimum void amount can be determined experimentally by subjecting the bed to a fluid flow and measuring the bed height at minimal fluidization L_{mf} [m] and comparing this value to the height of the bed when no liquid is flowing (where the porosity or void fraction is known or can be measured easily).

As previously stated, the pressure drop increases as the fluid velocity increases until the minimum fluidization begins. If the velocity increases further, the pressure drop decreases very slightly and then remains practically unchanged as the bed continues to develop height or as the porosity increases with velocity. The bed will finally look like a boiling liquid. When the bed expands with increasing fluid velocity, it should always have a horizontal upper surface. Finally, when the velocity is too fast, particles can be drawn into the flow and the bed can completely disintegrate.

The relationship between the bed height L and porosity ε for a bed with a uniform cross-section A is defined as follows: as the volume $(1 - \varepsilon)LA$ is equal to the total volume of solids if they existed as a single object:

$$L_1 A (1 - \varepsilon_1) = L_2 A (1 - \varepsilon_2) \quad (\text{eq.27})$$

$$\frac{L_1}{L_2} = \frac{(1 - \varepsilon_2)}{(1 - \varepsilon_1)} \quad (\text{eq.28})$$

where L_1 is the height of the bed with porosity ε_1 and L_2 is the height with porosity ε_2 .

2.4.2. Pressure drop and minimum fluidization velocity.

As a first approximation, the pressure drop at the start of fluidization can be determined as follows. The force obtained from the pressure drop multiplied by the cross-sectional area shall be equal to the gravitational force exerted by the mass of the particles minus the buoyancy of the displaced liquid.

$$\Delta p A = L_{mf} A (1 - \varepsilon_{mf}) (\rho_p - \rho) g \quad (\text{eq. 29})$$

So:

$$\frac{\Delta p}{L_{mf}} = (1 - \varepsilon_{mf}) (\rho_p - \rho) g \quad (\text{eq. 30})$$

Often, we have an irregular shape of particles in the bed and it is more convenient to use particle size and shape factor in the equations. We will first replace the mean effective diameter D_p by the term $\phi_s D_p$ where D_p now represents the particle size of a sphere having the same volume as the particle and ϕ_s the shape factor. Finally, equation 22 for the pressure drop in a packed bed becomes:

$$\frac{\Delta p}{L_l} = \frac{150 \mu v' (1-\varepsilon)^2}{\Phi_s^2 D_p^2 \varepsilon^3} + \frac{1.75 \rho (v')^2 (1-\varepsilon)}{\Phi_s D_p \varepsilon^3} \quad (\text{eq. 31})$$

where L_l = the bed height [m].

Equation 31 can now be used for extrapolation of packed beds to calculate the minimum fluid velocity v'_{mf} at which fluidization begins by substituting v'_{mf} for v' , ε_{mf} for ε , and L_{mf} for L_l and combining the result of equation 30 to give:

$$\frac{1.75 D_p^2 (v'_{mf})^2 \rho^2}{\phi_s \varepsilon_{mf}^3 \mu^2} + \frac{150 (1-\varepsilon_{mf}) D_p v'_{mf} \rho}{\phi_s^2 \varepsilon_{mf}^3 \mu} - \frac{D_p^3 \rho (\rho_p - \rho) g}{\mu^2} = 0 \quad (\text{eq. 32})$$

By defining the Reynolds number as:

$$N_{Re,mf} = \frac{D_p v'_{mf} \rho}{\mu} \quad (\text{eq. 33})$$

Equation 33 becomes:

$$\frac{1.75 (N_{Re,mf})^2}{\phi_s \varepsilon_{mf}^3} + \frac{150 (1-\varepsilon_{mf}) (N_{Re,mf})}{\phi_s^2 \varepsilon_{mf}^3} - \frac{D_p^3 \rho (\rho_p - \rho) g}{\mu^2} = 0 \quad (\text{eq. 34})$$

When $N_{Re, mf} < 20$ (small particles), the first term of equation 34 can be neglected and when $N_{Re, mf} > 1000$ (large particles), the second term can be neglected.

If the terms ε_{mf} and/or ϕ_s are not known, Wen and Yu have found for a variety of systems:

$$\phi_s \varepsilon_{mf}^3 \cong \frac{1}{14}, \quad \frac{1-\varepsilon_{mf}}{\phi_s^2 \varepsilon_{mf}^3} \cong 11 \quad (\text{eq. 35})$$

Substituting in equation 34, we get another empirical equation to be used only when absolutely necessary:

$$N_{Re,mf} = \left[(33.7)^2 + 0.0408 \frac{D_p^3 \rho (\rho_p - \rho) g}{\mu^2} \right]^{1/2} - 33.7 \quad (\text{eq. 36})$$

This equation is valid for a large range of Reynolds numbers from 0.001 to 4000 with an average deviation of 25%.

2.4.3. Expansion of fluidized beds.

In the case of small particles and where $N_{Re,mf} = \frac{D_p v' \rho}{\mu} < 20$, we can estimate the variation of the porosity or L the bed height as follows. We assume that equation 34 applies over the entire range of fluid velocities with the first term neglected. Next, the solution for v' is:

$$v' = \frac{D_p^2 (\rho_p - \rho) g \phi_s^2}{150 \mu} \frac{\varepsilon^3}{1 - \varepsilon} = K_1 \frac{\varepsilon^3}{1 - \varepsilon} \quad (\text{eq. 37})$$

We note that all terms except ε are constant for the particular system and v' depends on ε . This equation can be used with liquids to estimate porosities of the order of $\varepsilon < 0.80$. However, due to agglutination and other factors, errors can occur when using a gas.

3. Experimental part

3.1 Objectives

Estimate the particle diameter of an unknown sample in a bed based on pressure loss.

Estimate the size distribution of particles in a mixed sample.

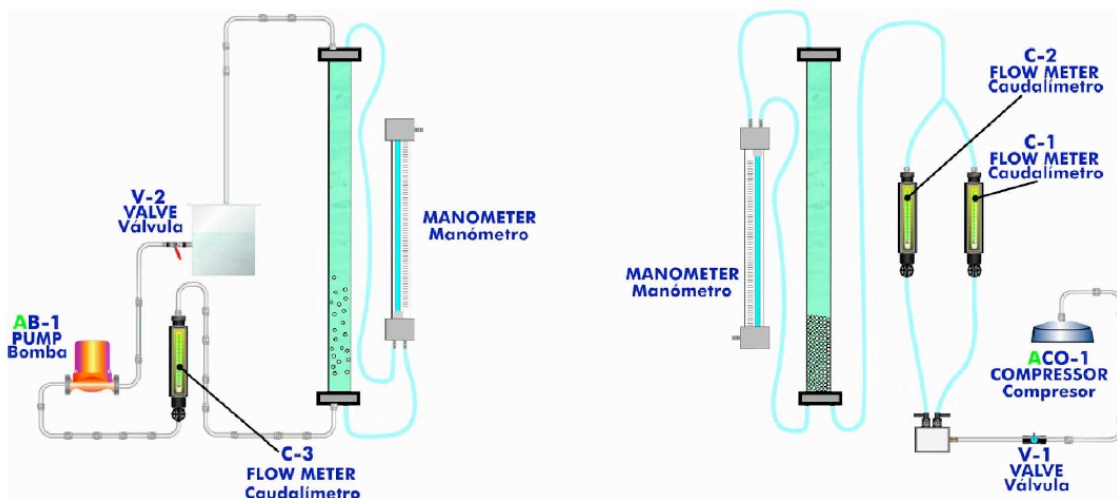
Measure the minimum fluidization flow rate.

Estimate particle density based on these observations.

Design a large-scale column according to the needs of the company.

3.2 Experimental set-up

Given the importance of your task, a pilot plant has been set up to enable you to calculate the pump capacity according to the following diagram:



The columns are 550 mm high with an internal diameter of 44 mm.

Five particle samples are available for this experiment:

No.4 - D_p = unknown

No.5 - D_p = 2 mm

No.6 - D_p = 3 mm

No.7 - D_p = 4 mm

No.8 – Mix of particles with two different diameters

3.3 Experimental procedure

Water fluidized bed

Before starting the system, measure the porosity of the particles used in the fluidized bed with water. Use the volumetric cylinder with the particles, water, and the balance (to measure the mass) in a way that allows you to calculate the porosity of the bed. Is there any other way to measure the void fraction?

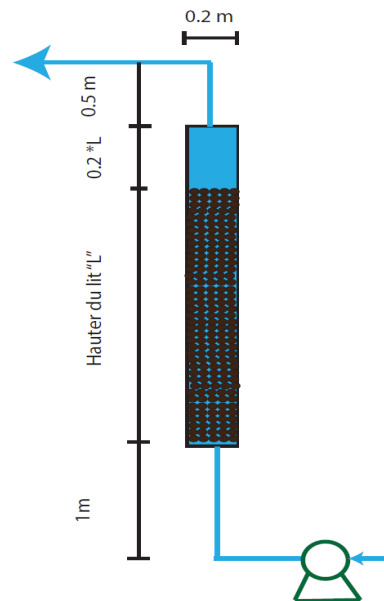
- 1) Turn on the device, start the software, and name your file.
- 2) Using the samples with a known particle diameter (three samples indicated by the instructor) check the validity of Ergün's equation (eq.22) by comparing the variation of pressure losses as a function of water flow: (Show on a graph in your report).
- 3) For samples of unknown particle size: Record the bed height without flow, start the pump (AB-1) and by increasing the water flow in the bed observe the variation of the pressure loss as a function of the flow rate. At what flow rate do you observe the fluidization?
- 4) Same as 3) but for a sample given by the instructor containing various proportions of particles with two different diameters.
- 5) By plotting the pressure drop (Δp) as a function of v' calculate the particle diameter for unknown samples. What assumption is made using this specific equation? Find the minimum fluidization velocity and find the density of the particles.
- 6) Qualitatively investigate the fluidization of the particle bed with air as indicated by the instructor (no data acquisition required).

Report

- 1) Graphically represent Δp as a function of v' and report the properties of the particles found. For particles of known diameter, is the pressure drop, Δp , predicted by equation 22 consistent with the experimental data? What could be the cause(s) of eventual deviations? Compare these on the same graph. What is the average size of the particles in the mixed sample?
- 2) Knowing the density of the particles used and a bed particle mass of 200 kg:
 - Calculate the volume of the bed and column (20% larger of the bed) according to the figure below.
 - Calculate the minimum fluidization velocity using the minimum fluidization porosity (ϵ_{mf}) calculated in the experimental setup and knowing that the

minimum fluidization bed height (L_{mf}) is related to the change recorded in the experiment. The shape factor is fixed at 1.

- Calculate the pressure drop in the bed (Δp)
- Calculate the water flow rate and size the pump required (kW).



- 3) How do you think the pressure drop will change when larger particles are used in your water fluidized bed? What is the effect on pump power?
- 4) The company also wants you to evaluate the consequences of changing the shapes of the particles. Can you postulate what would be the effect(s) of using cubic particles? What about if we use cylindrical particles with equivalent length and diameter? Assume that the cubic and cylindrical particles have the same volume and porosity than the spherical particles.
- 5) Discuss the pros and cons of using large particles in the bed? What about small particles?
- 6) At the coffee machine, one of your colleagues suggest that you could flow water from the top of the column to reduce the power required by the pump to fluidize the particles. What do you think about this idea?